Anomalous acoustic reflection on a sliding interface or a shear band

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We study the reflection of an acoustic plane wave from a steadily sliding planar interface with velocity-strengthening friction or a shear band in a confined granular medium. The corresponding acoustic impedance is utterly different from that of the static interface. In particular, the system being open, the energy of an in-plane polarized wave is no longer conserved, the work of the external pulling force being partitioned between frictional dissipation and gain (of either sign) of coherent acoustic energy. Large values of the friction coefficient favor energy gain, while velocity strengthening tends to suppress it. An interface with infinite elastic contrast (one rigid medium) and v-independent (Coulomb) friction exhibits spontaneous acoustic emission, as already shown by Nosonovsky and Adams [Int. J. Eng. Sci. **39**, 1257 (2001)]. But this pathology is cured by a moderately large V strengthening of friction, or, for systems with not too large friction coefficients, by any finite elastic contrast. We show that (i) positive gain should be observable for rough-on-flat multicontact interfaces and (ii) a sliding shear band in a granular medium should give rise to sizable reflection, which opens a promising possibility for the detection of shear localization.

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I. INTRODUCTION

The question of the origin and nature of shear localization in disordered systems, such as soft glasses or confined granular media, which are jammed at equilibrium, but flow when sheared beyond a threshold stress, is a long standing one. Due, in particular, to recent progress in theoretical descriptions, it is presently the subject of renewed interest. Hence the need for identifying appropriate, noninvasive methods of experimental investigation which could complement the optical and NMR imaging ones, recently put to use by Pignon *et al.* [1], and by Coussot *et al.* [2]. We intend to show in this paper that the propagation of sound pulses appears as a promising possibility. Namely, we will show that the presence of a shear band of thickness small compared with the acoustic wavelength should give rise to strong anomalous reflection of a transverse acoustic signal for well defined incidence ranges. Such a method could therefore provide a relatively handy fingerprint of shear localization in confined granular media.

An extreme case of localized shear flow is that of frictional sliding of the interface between two cohesive macroscopic solids. In such a configuration, the very structure of the system prelocalizes shear to the nanometer-thick layer which forms the molecular adhesive contact(s). The role of the above-mentioned threshold stress is played here by the so-called static friction force. Below this threshold, the interface is elastically pinned (jammed), and responds elastically to a shearing force. So, the corresponding mechanical boundary condition is simply that the displacement fields in the two media must be fully continuous across the interface. But, beyond this shear level, sliding sets in and, along the direction of relative motion, the boundary condition is now provided by the dynamic friction law, which states that the shear and normal stresses are proportional. Obviously, such a discontinuity of the boundary conditions must result in a discontinuous change of the acoustic reflection and transmission coefficients when the system is set into sliding.

This was pointed out already long ago by Chez *et al.* [3], who studied the reflection of a sound wave propagating in a plane orthogonal to the sliding direction, and polarized in the plane of incidence on an interface with the Coulomb frictional behavior (constant friction coefficient). However, due to the choice of this particular geometry, they overlooked an interesting effect. Indeed, the sliding system is an open one: energy is being pumped in from an external source—the external driving machine which imposes the sliding velocity. So, as soon as the acoustic displacement field has a nonzero component, in the interfacial plane, along the direction of the sliding motion, additional mechanical work (of *a priori* either sign) is extracted from the external source, and interfacial acoustic scattering is no longer energy-conserving. This opens, in principle, the possibility of acoustic gain at reflection, i.e., conversion of incoherent into coherent mechanical energy—quite an exciting prospect indeed.

Now, from the point of view of the propagation of an acoustic signal of wavelength λ, a shear band of thickness $d \ll \lambda$ in a confined granular medium appears as the equivalent of a frictional interface between two identical solids. Indeed, the band can then be considered as a surface of mechanical discontinuity between the nonsliding adjacent regions, which behave as (disordered) elastic solids. Experimental studies by rock mechanicians [4,5] of systems constituted of two bulk rock pieces separated by an interposed layer of granular material (called ‘‘gouge’’) have established that in such systems (i) sliding occurs in a narrow band within the gouge and (ii) the dynamics is ruled by a standard solid friction law, the associated friction coefficients having a magnitude comparable with those for solids in direct contact.

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Reflection and transmission of waves with a polarization component along the sliding direction of an interface with constant friction coefficient between dissimilar media have been recently studied by Nosonovsky and Adams [6], though in a different perspective. Namely, they focused primarily on the possible generation of slip pulses—a dynamical feature that seems to be specific of the pure Coulomb friction. In the course of this paper we will rederive some of their results, which will be extended to the more realistic case of velocity-dependent friction and to the shear band problem.

This paper is organized as follows.

In Sec. 2, we first write down the equations for the most general case of a monochromatic acoustic wave incident upon a planar frictional interface between two semi-infinite solids with different elastic moduli. We then specialize in Sec. III to the case where the elastic contrast between the two media is very large (e.g., a gel sliding on top of a glass plate). We show that, if the stiffer medium is assumed strictly nondeformable and friction taken to be Coulomb-like, the reflection coefficient of a wave with the sliding direction and polarization in the plane of incidence is highly pathological: not only is a huge gain at reflection possible for some particular incidence range, but spontaneous acoustic emission from the surface is predicted—a result already obtained recently by Nosonovsky and Adams [7]. These singularities are to be related to those already found by Adams [8], and Ranjith and Rice [9], in their studies of interfacial waves in the same system. They result, as is well known in mechanics, from the specific singular character which the Coulomb model, which takes the friction coefficient to be a mere constant, shares with the Hill model of plasticity. Indeed, we show that (i) a very small finite relative compliance of the stiffer medium is sufficient to destroy the acoustic emission singularity and (ii) improving upon the Coulomb description by taking into account a velocity-strengthening dependence of the dynamic friction coefficient of the order of what is measured for real systems also cures this singularity. Moreover, the possibility of energy gain at reflection is found to be strongly dependent upon the strength of the velocity dependence, and hence on the type of system: while it should be negligible for a gel-on-glass system, it might be observable with a rough-on-flat multicontact interface in well defined incidence ranges.

Section IV is devoted to the symmetric case of two mechanically identical solids relevant to the shear band problem. In this situation, and when the velocity dependence of the friction coefficient is taken into account, we predict that the sliding shear band should give rise to a clear acoustic signature—namely, a reflection coefficient with magnitude typically of the order of unity for incident signals in well defined ranges of incidence angles.

II. GENERAL FORMULATION

Consider two elastically isotropic semi-infinite media M and M' with shear moduli and Poisson ratios $(\mu,\nu;\mu',\nu')$ and densities (ρ,ρ'), occupying, respectively, (Fig. 1) the upper $(x_2>0)$ and lower $(x_2<0)$ half spaces, and in continuous contact with each other along the $x_2=0$ plane.

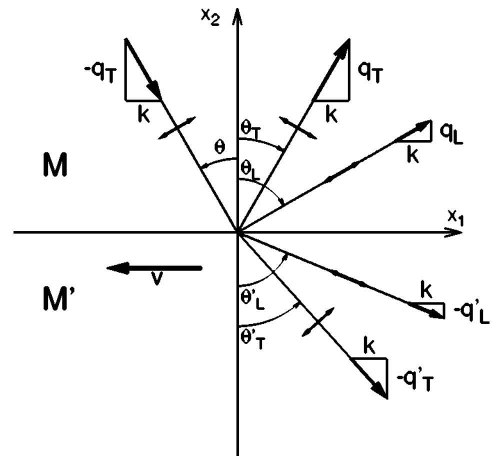

FIG. 1. Schematic representation of the system: a transverse incident wave impinges at incidence θ onto the sliding interface, giving rise to two reflected and two transmitted waves.

Medium M' is assumed to be in a stationary sliding motion with respect to medium M at velocity v along x_1 and towards $x_1<0$. In this reference state, the (homogeneous) normal and shear stresses τ_{22}^* and τ_{12}^*, are related by the dynamic friction law

$$\tau_{12}^* = -f(v)\tau_{22}^*, \tag{1}$$

with $f(v)$ the friction coefficient.

An emitter linked to medium M is sending from infinity towards the interface a plane acoustic wave of frequency ω, propagating in the x_1,x_2 plane at incidence angle θ (Fig. 1) and polarized in the plane of incidence. That is, the associated displacement has a finite component along the sliding direction. In order to fix ideas, and for the sake of simplicity, we restrict in most of what follows the algebraic formulation to the case of a transverse (shear) incident wave—the case of an incident longitudinal (dilatational) signal follows straightforwardly. The elastic displacement field (u_1,u_2) in medium M obeys the Lamé equations

$$\ddot{u}_1 = c_L^2\frac{\partial^2 u_1}{\partial x_1^2} + (c_L^2 - c_T^2)\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + c_T^2\frac{\partial^2 u_1}{\partial x_2^2}, \tag{2}$$

$$\ddot{u}_2 = c_L^2\frac{\partial^2 u_2}{\partial x_2^2} + (c_L^2 - c_T^2)\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + c_T^2\frac{\partial^2 u_2}{\partial x_1^2}, \tag{3}$$

c_L, c_T are the longitudinal and transverse sound velocities in medium M, and

$$c_T^2 = \frac{\mu}{\rho}, \qquad \frac{c_L^2}{c_T^2} = \frac{2(1-\nu)}{1-2\nu}. \tag{4}$$

The solution of Eqs. (2) and (3) reads

$$u_1 = \epsilon e^{i(kx_1-\omega t)}[e^{-iq_T x_2} + \alpha e^{iq_T x_2} + \beta e^{iq_L x_2}], \tag{5}$$

$$u_2 = \epsilon e^{i(kx_1-\omega t)}\left[\frac{k}{q_T}e^{-iq_Tx_2} - \alpha\frac{k}{q_T}e^{iq_Tx_2} + \beta\frac{q_L}{k}e^{iq_Lx_2}\right], \tag{6}$$

where ϵ specifies the amplitude of the incident wave, k is the x_1 component of its wave vector, and

$$q_{T,L}^2 = \frac{\omega^2}{c_{T,L}^2} - k^2, \tag{7}$$

with q_T real positive and $\mathrm{Re}\, q_L \geqslant 0$ and $\mathrm{Im}\, q_L \geqslant 0$.

In the lower medium M', which moves at velocity $-v$ in our reference frame, analogously,

$$u_1' = \epsilon e^{i(kx_1-\omega t)}[\alpha' e^{-iq_T' x_2} + \beta' e^{-iq_L' x_2}], \tag{8}$$

$$u_2' = \epsilon e^{i(kx_1-\omega t)}\left[\alpha'\frac{k}{q_T'}e^{-iq_T'x_2} - \beta'\frac{q_L'}{k}e^{-iq_L'x_2}\right], \tag{9}$$

where

$$\omega' = \omega + vk, \quad {q'_{T,L}}^2 = \frac{{\omega'}^2}{{c'}_{T,L}^2} - k^2. \tag{10}$$

In order to determine the amplitude reflection (α,β) and transmission (α',β') coefficients into the transverse and longitudinal channels, we must now specify the four boundary conditions (BC's) to be satisfied along the *deformed* M/M' interface.

Since the acoustic stresses we consider are small, $O(\epsilon)$, we expect the contact to persist everywhere, so that (i) normal displacements on both sides of the interface are equal, $\boldsymbol{u}\cdot\hat{\boldsymbol{n}} = \boldsymbol{u}'\cdot\hat{\boldsymbol{n}}$, and mechanical equilibrium imposes continuity of normal and shear stresses (ii) $\tau_{nn} = \tau'_{nn}$, (iii) $\tau_{nt} = \tau'_{nt}$. The sliding velocity perturbation $(v_I - v)$ being also $O(\epsilon)$, v_I remains positive (sliding persists) everywhere, and we assume that the dynamic friction condition now holds *locally*, namely that, at each interfacial point (iv) $\tau_{nt} + f(v_I)\tau_{nn} = 0$.

These boundary conditions, which have been taken for granted in existing studies of interfacial waves and shear fractures, actually imply important physical assumptions, which should be made explicit.

On the one hand, the (macroscopic) contact is supposed to be continuous and homogeneous—hence the statement of homogeneous stresses in the reference state. Now, it is well known that such is not the prevalent case. Most interfaces, being formed between rough solids, are actually constituted of a large, but rather sparse, set of microcontacts [10,11], in between which the surfaces are mechanically free. Then, it is clear that conditions (i)-(iii) are valid only in a coarse-grained sense, i.e., provided that the length scale $2\pi/k$ of the variation of the acoustic fields along x_1 is much larger than the average distance d between microcontacts, that is, for $kd \ll 1$.

Intercontact distances lie commonly in the range of hundreds of micrometers. Only in the case where at least one of the solids is extremely compliant are macroscopic contacts truly continuous. This is, for example, the case for elastomers or gels.

On the other hand, the mere existence of a finite static threshold proves that frictional dissipation results from the triggering by the applied shear of structural instabilities. It is now documented that the corresponding structural rearrangement events take place in the nanometer-thick adhesive interfacial layers, and affect volumes of, typically, nanometric scale [11,12]—comparable with the "shear transformation zones" invoked by Falk and Langer [13] to model the plasticity of amorphous solids. Let us call b the size of such a zone. A friction law represents the result of a *statistical average* over a large number of such dissipative events. So, it can only make sense on a scale much larger than b, that is for $kb \ll 1$.

All this means that boundary conditions (i)–(iv) above must be understood as valid only on scales larger than some *cutoff length* $L = \max(b,d)$ of the order of, typically, (i) a fraction of micrometer for conforming solid contacts, (ii) a fraction of millimeter for the, more common, multicontact interfaces.

Finally, since v_I now becomes a time-dependent quantity, condition (iv) implies that we assume the friction relation to hold instantaneously on the scale ω^{-1}. It is known that this does not apply to the case where the steady state $f(v)$ is velocity weakening $(df/dv < 0)$. Indeed, such behavior necessarily results from the action of some underlying structural dynamics leading to aging when sticking and rejuvenation upon sliding, such as that associated with the slow creep growth of the real area of contact relevant to multicontact interfaces [14,15]. Then, the equations describing nonsteady friction must involve explicitly at least one more dynamical "state" variable, and condition (iv) becomes insufficient. Note also that it is in this regime that steady sliding may be unstable with respect to stick slip.

So, we restrict ourselves in what follows to the velocity-strengthening case $df/dv > 0$. This can be expected to hold, for rough-on-rough interfaces, only at sliding velocities in the mm $\sec^{-1}$ range, large enough for rejuvenation effects to be saturated [16]. However, it has been shown [17] to prevail down to the μm $\sec^{-1}$ range when working with rough-on-flat interfaces where contacts keep their identity when sliding, which makes contact area saturation easily realizable.

The position of our deformed interface is given by

$$x_{2I}(x_1,t) = u_2(x_1 - u_1(x_1,0,t)). \tag{11}$$

We assume from now on that acoustic deformations are small enough for us to work in the linearized approximation. Then, the normal and tangent unit vectors $\hat{\boldsymbol{n}}, \hat{\boldsymbol{t}}$ are simply

$$\hat{\boldsymbol{n}}(x_1,t) = \begin{pmatrix} -\left(\dfrac{\partial u_2}{\partial x_1}\right)_{x_1,0,t} \\ 1 \end{pmatrix}, \quad \hat{\boldsymbol{t}}(x_1,t) = \begin{pmatrix} 1 \\ \left(\dfrac{\partial u_2}{\partial x_1}\right)_{x_1,0,t} \end{pmatrix}. \tag{12}$$

Let the stress field in, say, medium M, be denoted $\tau_{ij}^* + \delta\tau_{ij}$ $(i,j=1,2)$, with

$$\delta\tau_{ij}=\mu\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}+\frac{2\nu}{1-2\nu}\delta_{ij}u_{kk}\right]. \tag{13}$$

Then, to first order, at the interface in medium M,

$$\tau_{nn}=n_i\tau_{ij}n_j=\tau_{22}^*+\delta\tau_{22}-2\tau_{12}^*\left(\frac{\partial u_2}{\partial x_1}\right), \tag{14}$$

$$\tau_{nt}=\tau_{12}^*+\delta\tau_{12}+(\tau_{22}^*-\tau_{11}^*)\left(\frac{\partial u_2}{\partial x_1}\right) \tag{15}$$

and the local sliding velocity

$$v_I=v+(\dot{u}_1-\dot{u}_1'). \tag{16}$$

Conditions (i)–(iv) then become

$$[u_2-u_2']_{x_1,0,t}=0, \tag{17}$$

$$\left[\delta\tau_{22}-\delta\tau_{22}'-2\tau_{12}^*\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_2'}{\partial x_1}\right)\right]_{x_1,0,t}=0, \tag{18}$$

$$\left[\delta\tau_{12}-\delta\tau_{12}'+(\tau_{22}^*-\tau_{11}^*)\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_2'}{\partial x_1}\right)\right]_{x_1,0,t}=0, \tag{19}$$

$$\left[\delta\tau_{12}+f(v)\delta\tau_{22}-\tau^*\frac{\partial u_2}{\partial x_1}+f'(v)\tau_{22}^*(\dot{u}_1-\dot{u}_1')\right]_{x_1,0,t}=0, \tag{20}$$

where $f'(v)=df/dv$, and we have set

$$\tau^*=-[\tau_{22}^*-\tau_{11}^*-2f(v)\tau_{12}^*], \tag{21}$$

$\tau_{22}^*<0$ (compressive normal stress), $\tau_{12}^*=-\tau_{22}^*>0$, and we can reasonably assume sliding to occur under zero lateral stress, i.e., $\tau_{11}^*=0$, so that, in general, $\tau^*>0$.

Note that the τ^*-dependent terms in Eqs. (17)–(20), which account for the fact that the BC's must be enforced along the *deformed* interface, have usually been overlooked in previous works on interfacial waves. As will appear below, they give rise to corrections of the order of (τ^*/μ). In the case of hard materials, externally applied stresses are in practice always considerably smaller than elastic moduli, and these corrections can safely be neglected. However, such may not be the case when dealing with very compliant materials such as gels or elastomers. For example, for the case of gel-glass interfaces [18] sliding stress levels may be a sizable fraction of μ, and the full expressions should be retained. One can of course argue that nonlinear elasticity effects would also result in corrections of the order of (τ^*/μ), and should therefore be considered on the same footing. However, many very compliant materials are known to be linear up to very large deformations—e.g., $\simeq 0.4$ for the gelatin gels studied in Ref. [18]. For these materials the above-mentioned corrections should thus be relevant.

Then, using Eqs. (5), (6), (8), (9), and (13), the BC's yield the following set of four non homogeneous linear equations for $\alpha,\alpha',\beta,\beta'$:

$$-\frac{k}{q_T}\alpha+\frac{q_L}{k}\beta-\frac{k}{q_T'}\alpha'+\frac{q_L'}{k}\beta'=-\frac{k}{q_T}, \tag{22}$$

$$-2\mu\alpha+\mu\frac{q_T^2-k^2}{k^2}\beta+2\mu'\alpha'-\mu'\frac{q_T'^2-k^2}{k^2}\beta'=2\mu, \tag{23}$$

$$\mu\frac{q_T^2-k^2}{q_T}\alpha+2\mu q_L\beta+\mu'\frac{q_T'^2-k^2}{q_T'}\alpha'+2\mu' q_L'\beta' =\mu\frac{q_T^2-k^2}{q_T}, \tag{24}$$

$$\left[\frac{q_T^2-k^2}{q_T}-2fk+\frac{\tau^*}{\mu}\frac{k^2}{q_T}\right]\alpha+\left[2q_L+f\frac{q_T^2-k^2}{k}-\frac{\tau^*}{\mu}q_L\right]\beta +\frac{Af\omega}{c_T}(\alpha+\beta-\alpha'-\beta') =\left[\frac{q_T^2-k^2}{q_T}+2fk+\frac{\tau^*}{\mu}\frac{k^2}{q_T}\right]-\frac{Af\omega}{c_T}, \tag{25}$$

where we have defined

$$A=\frac{f'(v)c_T}{f(v)}\frac{|\tau_{22}^*|}{\mu}, \tag{26}$$

which measures the dimensionless strength of the velocity-strengthening effect.

Equations (22)–(25) above are relevant to the case of a transverse incident wave. We will also display some results for a longitudinal one. In this latter case, the only modification concerns the first terms in the right-hand side (rhs) of Eqs. (5) and (6) which give the expression of the total acoustic displacement field. This results in leaving the left hand sides of the final equations (22)–(25) unchanged, only the rhs's being modified.

Since the general solution of our problem depends on a large number of parameters (four elastic moduli, two densities, the angle of incidence, the friction coefficient and its derivative), it will be more illuminating to focus on a few simple cases, namely, that of very large elastic contrast (quasi-non-deformable medium M'), and the symmetric case of two identical materials, relevant to the problem of shear band detection. Finally, we will limit ourselves to the strongly subsonic sliding velocity regime compatible with experimental realization, namely, v of the order of millimeters per second at most. For this reason, we will systematically take, in all numerical calculations, $\omega'=\omega$. We have checked that this approximation is totally unimportant in all the cases considered below.

III. LARGE ELASTIC CONTRAST CASE

We consider here the case of maximum asymmetry, where medium M' is considerably stiffer than medium M. In order to disentangle the respective effects of the elastic contrast ratio $R=\mu'/\mu$ and of the velocity dependence of friction, we first treat the extra simple limit of a nondeformable medium M' and of the pure Coulomb friction, then examine how the results thus obtained are modified (i) when R is large but finite and (ii) when $f'(v)\neq 0$.

A. Nondeformable medium M' and the Coulomb friction

The boundary conditions reduce to imposing that $u_2=0$ and $\tau_{nt}+f\tau_{nn}=0$ along the $x_2=0$ plane. Equations (22)–(25) become, in this $R^{-1}=f'=0$ limit, where $\alpha'=\beta'=0$,

$$\frac{k}{q_T}\alpha-\frac{q_L}{k}\beta=\frac{k}{q_T}, \tag{27}$$

$$\left[\frac{k^2-q_T^2}{q_T}+2fk\right]\alpha+\left[-2q_L+f\frac{k^2-q_T^2}{k}\right]\beta=\frac{k^2-q_T^2}{q_T}-2fk, \tag{28}$$

which yield for the $T\rightarrow T$ and $T\rightarrow L$ reflection coefficients,

$$\alpha=1+\frac{4fq_L}{\Delta},\quad \beta=\frac{4fk^2}{q_T\Delta}, \tag{29}$$

where

$$\Delta=q_L\left(\frac{k}{q_T}+\frac{q_T}{k}\right)+f\left(q_T-2q_L-\frac{k^2}{q_T}\right). \tag{30}$$

Note that, for $f\rightarrow 0+$, and for incident waves propagating towards $x_1>0$ $(k>0)$, $\Delta>0$. That is, at least in the limit of weak friction and in this incidence range, clearly $\alpha>1,\beta>0$, and the reflected energy flux is larger than the incident one. In order to try and clear up this *a priori* surprising prediction, let us look in more detail into the question of energy balance in our system. Consider the volume V of a slice of unit depth along x_3 of medium M, limited by its area S $(-L<x_1<L)$ along the interface, and the semicylinder C_∞ of radius L $(L\rightarrow\infty)$. In the situation we are considering, of an incident wave of constant amplitude, the average over an acoustic period of the elastic energy stored within V is time independent. This means that energy conservation simply imposes that the net energy flux at infinity, $\dot{W}_\infty$, flowing out of C_∞, associated with the acoustic waves, must equal the increment associated with the acoustic perturbation of the work injected per unit time into the system via the work of the stresses acting on the moving interface, $\dot{W}_{ext}$, which is pumped from the driving machine. This is proved in detail in the Appendix, where we show that, in the present case of a nondeformable medium M', per unit area of S,

$$\langle\dot{W}_{ext}\rangle=\langle-\dot{u}_1\tau_{12}\rangle, \tag{31}$$

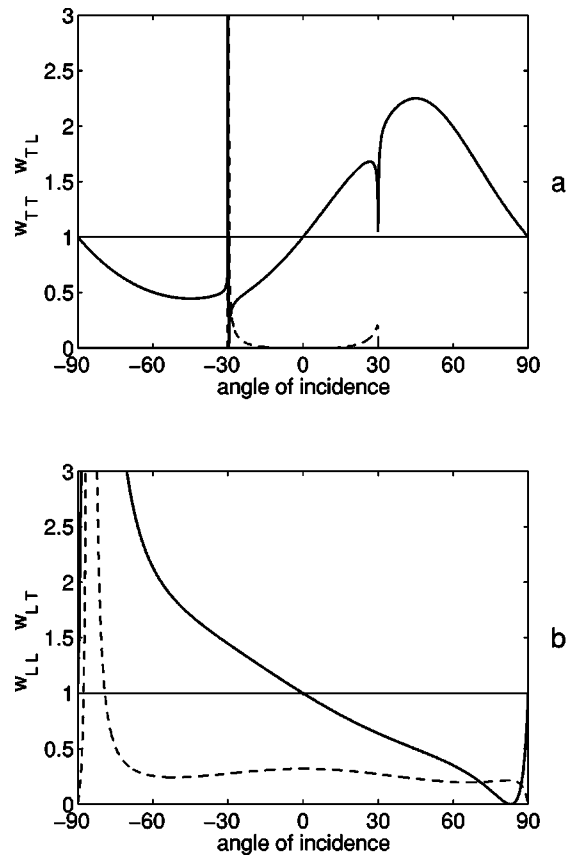

FIG. 2. Relative powers $w_{IJ}(\theta)(I,J=T,L)$ reflected from the interface between a compliant medium and a rigid one versus incidence angle θ (in degrees). Friction is velocity independent. $R=\infty$, $f=0.2$, $c_T/c_L=0.5$. Incident wave (a) transverse, w_{TT} (—), w_{TL} (- - -); (b) longitudinal , w_{LL} (—), w_{LT} (- - -).

where $\langle\cdots\rangle$ stands for the average over the acoustic period. We then easily prove (see the Appendix) that this is exactly equal to the net acoustic energy flux

$$\dot{W}_\infty=\langle\dot{W}_{refl}\rangle-\langle\dot{W}_{inc}\rangle. \tag{32}$$

In other words, a sliding frictional interface is, in principle, able to transform external incoherent mechanical work into coherent acoustic radiation, i.e., to act as an "acoustic laser." In order to make this more precise, we plot in Fig. 2(a) the relative powers reflected into the T and L channels, given by (see the Appendix)

$$w_{TT}=|\alpha|^2, \tag{33}$$

$$w_{TL}=\frac{q_Tq_L}{k^2}|\beta|^2 \quad \text{if } q_L \text{ is real},$$
$$=0 \quad \text{if } q_L \text{ is imaginary}, \tag{34}$$

against the incidence angle $\theta=\sin^{-1}(c_Tk/\omega)$ and for $f=0.2$, $c_T/c_L=0.5$. It is seen that, for all positive incidences, a transverse incident wave is predicted to be notably amplified upon reflection, while the power in the L channel remains quite small, even close to $\theta_{lim}=\sin^{-1}(c_T/c_L)$ beyond which the L wave becomes evanescent, where it is maximum. However, a much more surprising result, already derived by Nosonovsky and Adams [7] is that w_{TT} and w_{TL} are found to diverge at a negative incidence $-\theta_{cr}$, where the denominator Δ [Eq. (30)] of the expressions for α and β [Eq. (29)] vanishes. Using Eq. (7), the condition $\Delta=0$ reads

$$\sqrt{\frac{c_T^2}{c_L^2}-\sin^2\theta}=-\frac{f\cos 2\theta\sin\theta}{1-f\sin 2\theta}, \tag{35}$$

which is easily checked graphically to have a single negative solution $-\theta_{cr}$ with $\theta_{cr}<\theta_{lim}$, whatever the values of f and of c_T/c_L. The smaller the value of f, the closer θ_{cr} approaches θ_{lim}. In the case shown in Fig. 2(a), $\theta_{lim}-\theta_{cr}\simeq 7'$. In the case [Fig. 2(b)] of L incidence, the corresponding singularity occurs for θ close above $-90°$. In other words, in this admittedly oversimplified limit (rigid medium M', the pure Coulomb friction), homogeneous sliding at constant velocity is impossible: indeed, any infinitesimal perturbation is able to trigger the emission, all along the interface, of a coupled set of transverse and longitudinal acoustic waves, of *a priori* undetermined amplitude, traveling, respectively, at angles θ_{cr} and $\sin^{-1}[\sqrt{c_T^2/(c_L^2\sin^2\theta_{cr})-1}]$ in the back direction $-\hat{x}_1$. On the one hand this entails that an infinite external energy would be pumped in. On the other hand, as soon as the local interfacial velocity will vanish, the interface will stop (stick). So this pathologic behavior signals that homogeneous sliding is here absolutely unstable—a signature to be added to that provided by the existence in such systems of amplified interfacial waves, first identified and studied by Adams [8], which lead to the question of ill posedness of the problem of interfacial slip pulses studied by several authors and recently synthetized by Ranjith and Rice [9]. Whether or not the stable mode of sliding for our model interface would be a set of square slip pulses such as calculated in Ref. [7] is out of the reach of this work. We will rather concentrate on a different question, namely, how robust is this pathology? Does it persist in the presence of (i) a finite elastic compliance of the stiffer medium M' and (ii) a velocity-strengthening dependence of the friction coefficient? We now consider successively these two possibilities.

B. Finite elastic contrast and the Coulomb friction

Let us now consider the case of a large but finite elastic contrast ratio $R=\mu'/\mu\gg 1$. We assume for the sake of simplicity equality of the Poisson ratios $\nu=\nu'$. In this case, except for quasinormal incidence angles $q'^2_{T,L}<0$, there is total reflection at the interface, with only evanescent transmission into medium M'. α and β must now be obtained by solving the full Eqs. (22)–(25) for constant f. Except in the vicinity of $\theta=-\theta_{cr}$, a finite R^{-1} simply acts as a regular perturbation, leading to a correction $O(R^{-1})$. For $\theta\approx-\theta_{cr}$, however, one must consider in detail the behavior of the determinant $\mathcal{D}$ of system (22)–(25). To first order in R^{-1},

$$\mathcal{D}\propto\Delta(q_T,q_L)+R^{-1}\Gamma(q_T,q_L,q_T',q_L'). \tag{36}$$

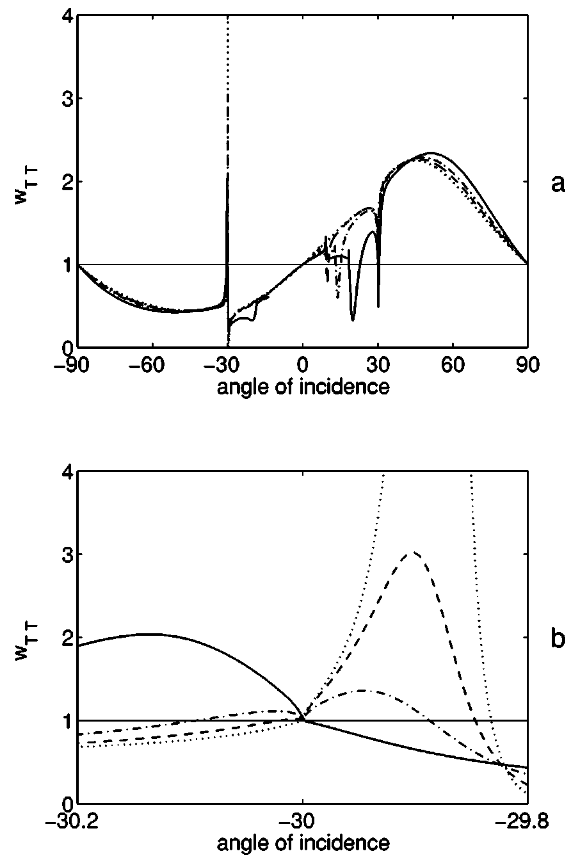

FIG. 3. Evolution of the relative reflected power $w_{TT}(\theta)$ with elastic contrast ratio R for the Coulombic friction. θ is measured in degrees, $f=0.2$, $c_T/c_L=0.5$, $R=\infty(\cdots)$, 40 (- - -), 20 (-·-·), 10 (—). (a) Full incidence range; (b) blowup for θ close to $-\theta_{lim}=-30°$.

As seen above, the zero of Δ always occurs for $\theta_{cr}<\theta_{lim}$ where q_L, and hence Δ, is real. On the other hand, it appears that, for $q'_{T,L}$ pure imaginary, Γ is a complex quantity. So, no zero of $\mathcal{D}$ which would evolve continuously from $-\theta_{cr}$ exists, and one finds that, for small but finite R^{-1}, α and β no longer diverge, but exhibit, in the vicinity of $-\theta_{cr}$, a maximum of order R: a finite compliance of medium M', however small it is, is sufficient to kill the pathology found in the totally rigid limit. This we have confirmed by computing the relative reflected powers $w_{IJ}(I,J=T,L)$ for various values of R and $f=0.2$. The results for w_{TT} are shown in Fig. 3: the smaller the value of R, the lower the maximum of w, which reduces to a few units for $R\lesssim 40$. Of course, in practice, for $R\gg 1$, the very large values of the reflection coefficients at maximum mean that a very small incident amplitude ϵ would suffice to induce a finite response, such that $v+\dot{u}_1$ would vanish, leading to interfacial stick—i.e., homogeneous slip, though no longer absolutely unstable from a strict mathematical point of view, would be very weakly stable, as it could not resist perturbations of finite but very small amplitude.

Let us, however, point out that the behavior depicted in Fig. 3 is specific of not too large friction coefficients only (in the case shown, $f=0.2$). Indeed, Nosonovsky and Adams [7] have found that the spontaneous emission pathology is present, for any given finite contrast R, provided that the friction coefficient exceeds a R-dependent threshold value [19] $f_{th}(R)$, which increases with the contrast from its minimum value $f_{th}(R=1)\simeq 0.48$, while exhibiting a divergent behavior for $R \gg 1$. For example, for $R\simeq 10$, $f_{th}>1$. Now, for a majority of wearless systems, friction coefficients in the low velocity regime (typically up to the mm/sec range) needed to perform well controlled acoustic reflection experiments do not exceed typically 0.5. For these, the above conclusions will be valid. If, however, $f \gtrsim 0.5$, the behavior is more complicated: the pathology is absent for $R>f_{th}^{-1}(f)$, but reappears for smaller elastic contrasts, in which case it becomes essential to look into the effect of the velocity dependence of friction.

C. Infinite elastic contrast and v-strengthening friction

The problem is now specified by Eqs. (22) and (24) in which $\alpha'=\beta'=0$, and the position of the singularity of α,β, if any, is therefore given by the zero of

$$\Delta_A = q_L\left(\frac{k}{q_T}+\frac{q_T}{k}\right)+f\left(q_T-2q_L-\frac{k^2}{q_T}\right) + A\,\frac{\omega f(v)}{c_T}\left(\frac{q_L}{k}+\frac{k}{q_T}\right), \qquad (37)$$

where the dimensionless parameter measuring the velocity-dependence effect is defined by Eq. (26):

$$A=\frac{f'(v)c_T}{f(v)}\frac{|\tau_{22}^{*}|}{\mu}.$$

When solving numerically for $\Delta_A=0$, we find that the singularity disappears for $A \geqslant A_{cr}$. More precisely, we find that, as A increases, the zero of Δ_A approaches $-\theta_{lim}$, which it reaches for $A=A_{cr}$, beyond which it disappears, due to the fact that no zero can occur in the regime where the L wave is evanescent. The threshold value A_{cr} is independent of the value of $f(v)$, but it depends upon the sound velocity ratio. For a reasonable choice of this parameter ($c_T/c_L<0.7$), A_{cr} is at most a few units. For example, for the presently used value $c_T/c_L=0.5$, we obtain $A_{cr}=1$. This behavior is illustrated in Fig. 4, where we plot the relative reflected power w_{TT} against incidence angle for various values of A. It is also seen that the larger the value of A, the smaller the gain at reflection. It appears that at another characteristic A value ($A=2$ for our parameter values), the gain becomes negative ($w_{TT}<1$) for all incidences. That is, a strenghtening v dependence of f is very efficient to kill the amplification effect.

It is therefore important to evaluate an order of magnitude of A for real interfaces with a large elastic contrast, a good example of which is that of gel-glass couples. Frictional sliding at the interface between glass and a 5% gelatin aqueous gel has recently been studied in detail by Baumberger *et al.* [18]. Using their data, we find that, for v in the mm sec^{-1} range, $f'(v)/f(v)\approx 2\times 10^3$ sec m^{-1}, while $(c_T|\tau_{22}^{*}|/\mu)\approx 2$ m sec^{-1}, so that $A\approx 4\times 10^3$ is a very large number.

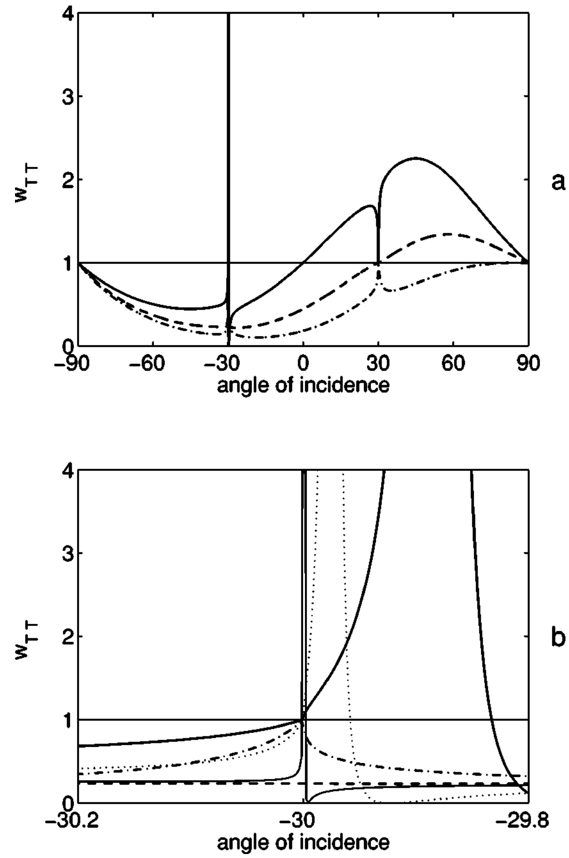

FIG. 4. Evolution of the relative reflected power $w_{TT}(\theta)$ with strengh A of the velocity dependence of the friction coefficient for rigid medium M'. θ is measured in degrees, $f=0.2$, $c_T/c_L=0.5$, $R=\infty$. $A=$ 0 (—), 1 (- - -), 2 (-·-·). (a) Full incidence range; (b) blowup for θ close to $-\theta_{lim}=-30°$. Two more curves, for $A=0.5$ (· · ·), 0.9 (thin full line), show the shift and narrowing of the singularity as $A=A_{cr}=1$ is approached. For $A \geqslant A_{cr}$, the peak amplitude becomes finite.

In this $A \gg 1$ limit, the rigid medium M' version of Eq. (24) reduces, to lowest order in A^{-1}, to

$$\alpha+\beta+1=0, \qquad (38)$$

that is, due to the high relative cost of increasing its instantaneous velocity, the interface remains locked to the homogeneous sliding state and $u_1\cong 0$.

So, for gel-glass sytems, acoustic reflection should not in practice be able to distinguish between the static and the sliding interfaces.

Another realization of the high elastic contrast situation is provided by the multicontact interface between rough glassy PMMA and atomically flat silanized glass, recently studied by Bureau *et al.* [17]. For this couple, $\mu'/\mu\simeq 20$. Thanks to the flatness of the glass surface, it is possible to saturate the slow growth of the real area of contact, which is responsible, in the case of rough-on-rough systems, for the velocity-weakening behavior of the dynamic friction coefficient and of the associated stick-slip dynamics. Then, for $v \gtrsim 1$ μm sec^{-1}, $f(v)$ is velocity strenthening and of the form

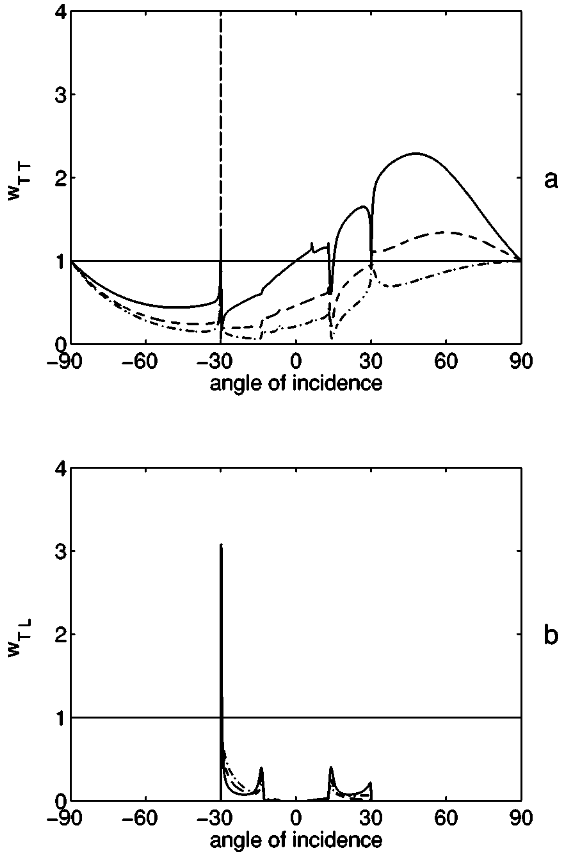

FIG. 5. Evolution of relative reflected powers with increasing A. Incident T wave, $R=20$, $f=0.2$, $c_T/c_L=0.5$. $A=$ 0 (—), 1 (- - -), 2 (-·-·). (a) $w_{TT}(\theta)$, (b) $w_{TL}(\theta)$. The height of the peak for $\theta=-\theta_{lim}$ is maximum, but finite, for $A=1$. θ is measured in degrees.

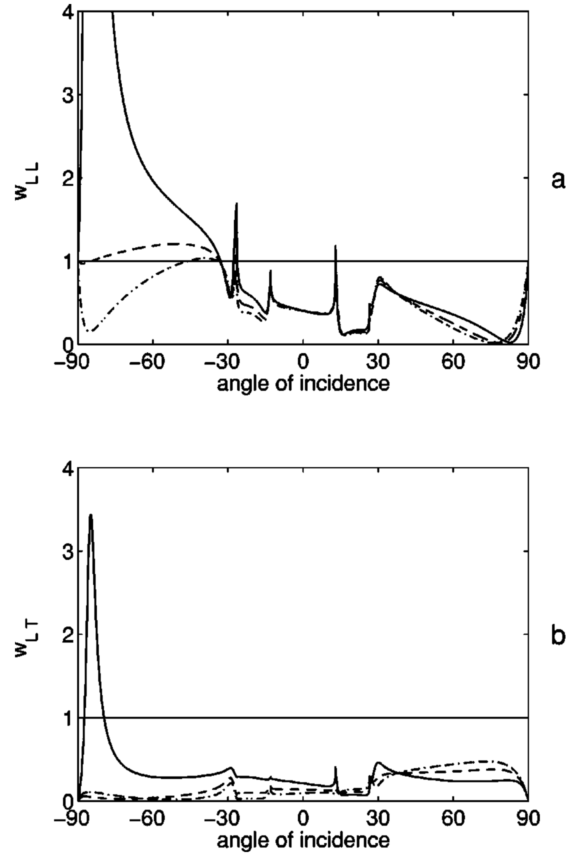

FIG. 6. Same plot as in Fig. 5, but for an incident L wave. (a) $w_{LL}(\theta)$, (b) $w_{LT}(\theta)$.

$$f(v)=f_0\left[1+\zeta\ln\left(\frac{v}{v_0}\right)\right], \tag{39}$$

where $f_0\approx 0.2$, and $\zeta\approx(2\text{–}4)\times 10^{-2}$. With $\mu_{PMMA}=1$ GPa, $c_T\simeq 10^3$ m sec^{-1}, and under normal stresses of the order of 5 kPa, one gets

$$A\approx\frac{100-200}{v_{\mu\text{m sec}^{-1}}}. \tag{40}$$

That is, for sliding velocities in the 100 μm sec^{-1} range, A is typically of the order of 1–2.

Finally, note that, since in this configuration the average distance between the micrometric regions which form the real area of contact is of the order of a fraction of millimeter, such an experiment would ask for acoustic signals in the range of a few hundred kHz at most, in order for our continuum description to be valid.

The relative powers reflected into the two channels for T and L incident waves under these conditions are plotted in Figs. 5 and 6, for $A=0,1,2$. (Since in this case typical values of τ^*/μ and v/c_T are $<10^{-5}$, calculations have been performed for $\tau^*=0$, $\omega'=\omega$.) Inspection of these results and comparison with the case of the nonmoving interface (Fig. 7) lead us to the following conclusions.

(i) The more favorable channels for observing gain at reflection are the diagonal (LL and TT) ones.

(ii) In both cases, a very narrow peak of w, reminiscent of the singularity obtained in the limit of the Coulomb friction on a rigid substrate, is predicted for $\theta\simeq-\theta_{lim}$. However, due to its narrowness, observing it would certainly be excessively demanding in terms of directional accuracy.

(iii) It thus appears more feasible to investigate one of the two following configurations: LL at large negative incidence angles, in the range of $-45°$, and TT at large positive incidence $\theta\approx 60°$.

(iv) Gain decreases very rapidly with increasing A, i.e., with decreasing v [see Eq. (40)]: while, for $A=1$ it reaches up to 20% in LL and 35% in TT, already for $A=2$ it has practically collapsed to zero. So, according to Eq. (40), the best situation corresponds to the largest possible driving velocities, in practice in the range of a fraction of mm/sec.

With these conditions in mind, it seems quite possible to obtain substantial acoustic gain at reflection on a sliding rough-on-flat multicontact interface.

IV. SYMMETRIC CASE

We now turn to the opposite limit where medium M and M' have the same elastic properties. It is *a priori* relevant to

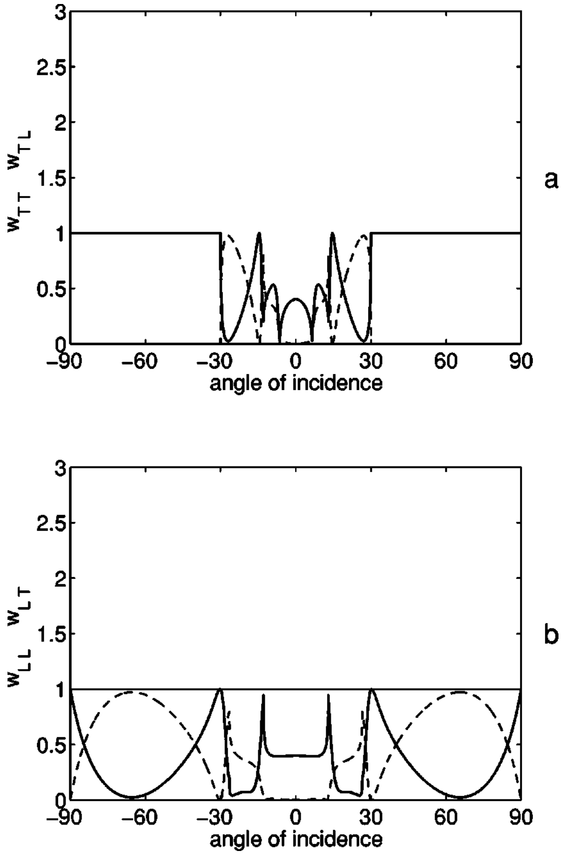

FIG. 7. Static interface: relative powers reflected from incident wave (a) transverse, w_{TT} (—), w_{TL} (- - -); (b) longitudinal, w_{LL} (—), w_{LT} (- - -). $R=20$, $c_T/c_L=0.5$. θ is measured in degrees.

the case of multicontact interfaces between two pieces of the same material. However, the above-mentioned rough-on-flat configuration is inappropriate in this case, since the asperities on the rough surface would give rise on the flat one to unavoidable and poorly qualified plastic damage and wear effects. On the other hand, as already mentioned, in the rough-on-rough configuration where such problems are irrelevant, geometric aging interrupted by motion results in a velocity-weakening behavior of $f(v)$ for velocities up to at least the mm $\sec^{-1}$ range, not very easy to access in a stationary sliding laboratory experiment. So, such symmetric solid on solid interfaces turns out not to be well adapted in practice to test our predictions.

The most interesting case, as already mentioned in Sec. I, is that of an established shear band, in particular, in a highly confined granular medium. In such highly disordered systems, internal stress inhomogeneities (the so-called stress-chain phenomenon) of course give rise to scattering of acoustic waves. However, this is all the smaller that the effective acoustic wavelength in the medium is larger with respect to the correlation length of stress fluctuations, known to be of the order of a few grain diameters D only. In this $kD \ll 1$ limit, as shown by experiments on the propagation of acoustic pulses [20], where one can separate out unambiguously the signal corresponding to the propagation of a ''coherent pulse,'' a description in terms of an effective acoustic medium is legitimate. Note that, as shear band thickness is also expected to be, for low shearing rates, of the order of a few D, $kD \ll 1$ also ensures that we can safely neglect it when dealing with our reflection-transmission problem.

Laboratory experiments on the frictional behavior of layers of granular rock confined between granite plates [4,5], and of glass bead assemblies [21] have shown that sheared confined granular media obey standard friction laws—namely, beyond a static threshold, $\tau_{12} = -f\tau_{22}$. However, the velocity dependence of the dynamic friction coefficient is not yet very clearly established. The data on rock point towards weakening at low velocities. However, they are strongly dependent upon the level of humidity. This, together with the very small size of the grains used in these experiments, strongly suggests that capillary condensation around the intergrain Hertz contacts is responsible for a slow strengthening of the medium, interrupted by sliding, which should become negligible either at low humidity or for larger grains. On the other hand, in the absence of such slow transients, since granular systems are essentially athermal, one expects logarithmic dynamic strengthening of the type found for multicontact interfaces [see Eq. (40)], which results from thermal activation effects [11,12], to be completely negligible. If such is the case, f would be v independent in the low velocity, quasistatic sliding regime. Note that this is what has been found by Géminard *et al.* [21] for glass beads completely immersed in water—though under relatively weak confinement. For this system, $f=0.2$. Preliminary results on highly confined glass beads [22] indicate $f \simeq 0.4$ and a very small velocity-strengthening effect, if any.

So, let us first consider the simplified case of the pure Coulomb friction $(A=0)$. We then solve the corresponding version of Eqs. (22)–(25) where we neglect τ^*/μ, since, for the confined granular systems made of hard materials (such as glass or steel) which we have in mind, under ordinary experimental conditions [20,23], this does not exceed, typically, 10^{-5} at most. Moreover, as we are interested here in strongly subsonic sliding velocities $(v/c_T < 10^{-5} - 10^{-6})$, $\omega' \simeq \omega$.

The results for the four relative reflected powers are plotted in Fig. 8 for the case $f=0.2$: while, in the TT channel, for negative θ there is loss at reflection, w_{TT} is seen to be sizable everywhere in the range $(-\theta_{lim};\theta_{lim})$ where a propagating L reflected wave exists, as well as for quasigrazing conditions. For $\theta = \pm\theta_{lim}$ the reflectance curve exhibits cusplike maxima—a standard behavior for multichannel scattering cross sections at the closing point of a channel. About $\theta = \theta_{lim}$, we even predict a gain of more than 50%, which becomes much larger for $f=0.4$. In the other three channels, w is much smaller—except, for the LL one, in quasigrazing conditions which are hardly realizable in practice.

This behavior is to be contrasted with what is expected from nonlocalized homogeneous shear sliding, namely, undisturbed propagation all through the sample. So, one expects the following to happen after starting to shear a confined sample at a constant rate, if a transverse acoustic pulse is sent into the granular pack in the direction normal to the shearing plane. At very short times, when the stress has not yet reached the sliding threshold, the system experiences uni-

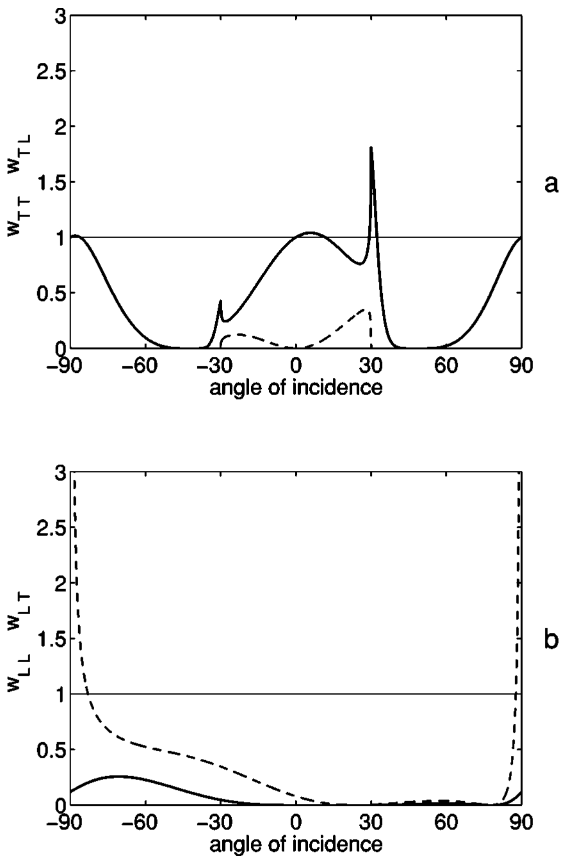

FIG. 8. Elastically symmetric case ($R=1$) and the Coulomb friction ($A=0$): relative powers reflected from incident wave (a) transverse, w_{TT} (—), w_{TL} (- - -); (b) longitudinal, w_{LL} (—), w_{LT} (- - -). $f=0.2$, $c_T/c_L=0.5$. θ is measured in degrees.

form elastic deformation, the incident pulse will give rise to a "coherent" reflected one, which will have traveled way and back *across the whole container*. After the initial sliding transient corresponding to the gradual installation of the shear band, during which fluctuations are expected to kill the coherent signal, *the reflected pulse should reappear, but with a distinctly shorter traveling time*, thus providing a clear signature of shear localization. Note that, since its traveling length is reduced, so will be the attenuation due to scattering by stress fluctuations, making it more easy to detect than the former signal. Finally, when sliding is stopped, the situation should revert practically to that before shearing, though the density contrast $\delta\rho/\rho$, of the order of a few percent at most, associated with the shear band [24] still persists. Indeed, then, $w_{TT}\approx(\delta\rho/\rho)^2(d\omega/c_T)^2$ is negligibly small, and pulse reflection only occurs, again, at the bottom end of the container.

The question is then to check how robust the reflectance characteristics predicted for $A=0$ are with respect to a possible, though probably small, velocity-strengthening dependence of the friction coefficient. It is seen in Fig. 9 that, although, as expected, w_{TT} decreases as A grows, at normal incidence it remains non-negligible up to the sizable value $A=2$, where it still is of the order of 25%. Again, in view of the results of Nosonovsky and Adams [7], a word of caution is in order. They predict that a symmetric system should be

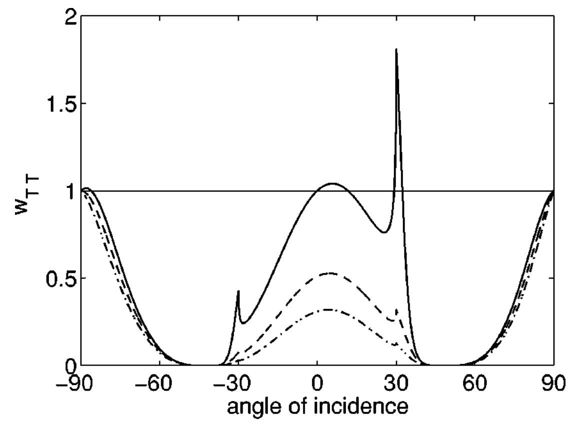

FIG. 9. Evolution of the relative reflected power $w_{TT}(\theta)$ with strengh A of the velocity dependence of the friction coefficient for symmetric case $R=1$. θ is measured in degrees, $f=0.2, c_T/c_L=0.5$. $A=$ 0 (—), 1 (- - -), 2 ($\cdot\cdot$-$\cdot$).

unstable against spontaneous emission for $f\gtrsim0.5$. If this happens to be the case for some granular materials, the question of the absence or presence of velocity strengthening, and of its magnitude, would become crucial: either A will be large enough to stabilize the steady sliding shear band, or up to now unsuspected effects (such as unsteady localization) might arise—a question which remains completely open for the moment.

V. CONCLUSION

In summary, from the above results, we conclude to the interest of performing experimental studies of the reflection of acoustic pulses on sliding solid interfaces and shear bands, the potential interest of which is different in each case.

As discussed in Sec. III C, experiments with a rough-on-flat multicontact interface open the possibility of obtaining acoustic gain, through the conversion by the frictional system of incoherent mechanical energy pumped from the driving system into coherent acoustic vibrations. However, such experiments are certainly delicate to realize, since they ask for working at non-normal incidences, and, due to the effect of velocity strengthening of dynamic friction coefficients, at not very small velocities, in the range of typically a few hundred μm/sec.

On the other hand, acoustic pulse reflection appears as a promising method for detecting shear localization in a confined granular medium. The best configuration—a transverse pulse at normal incidence—is more easily realizable, since it may be implemented with a single transducer acting as both emitter and receiver. Moreover, the expected signature (a strongly reduced transit time before the return of the reflected pulse) should be easy to identify—its presence confirming at the same time the expected quasiindependence of the sliding stress on the sliding velocity.

We therefore strongly hope that this work will motivate such experiments in the near future.

ACKNOWLEDGMENT

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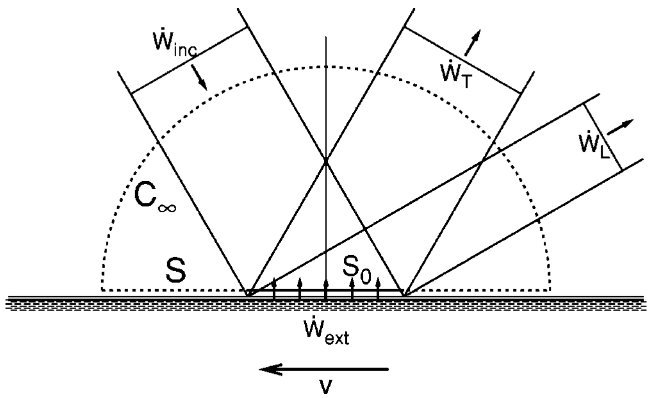

FIG. 10. Energy flows for rigid medium M'.

APPENDIX: ENERGY BALANCE AT A FRICTIONAL INTERFACE

We restrict ourselves to the case of a rigid substrate with an infinite elastic contrast $R=\mu'/\mu\to\infty$ considered in Sec. III A. This situation is sketched in Fig. 10, which concerns specifically the case of a transverse wave incident below the critical angle θ_{lim}, so that also the longitudinal reflected wave is propagating. In our two-dimensional geometry, the incident beam irradiates a strip S_0 of the interface perpendicular to the sliding direction x_1 and extending from $-L_0$ to L_0 along this direction. It has an area S_0 per unit length along x_3. The reflected beams stem from the irradiated area. The beams are wide enough to make the fringe effects negligible and easy to exclude as additive constants to the principal quantities proportional to S_0. The S_0 strip is now overlapped by a wider region S stretching from $-L$ to L such that a semicylindrical dome C_∞ raised over it encloses the regions of beam overlap and interference (as well as the fields of attenuated waves, should they arise). The bottom of the dome is infinitesimally above the interface, so that the equations of motion (2) and (3) are valid both inside and on the surface of the region V enclosed by the dome and the bottom plane. Then, the total elastic energy W inside the dome can be studied using the differential energy conservation law [25,26] (see also Ref. [3])

$$\mathrm{div}\mathbf{J}+\dot{e}=0, \quad \text{i.e.,} \quad \mathrm{J}_{\ell,\ell}+\dot{e}=0, \tag{A1}$$

where e is the local energy density, whose form will not be needed, and the energy current density $\mathbf{J}$ is given by

$$\mathbf{J}=-\dot{\boldsymbol{u}}\cdot\boldsymbol{\tau}, \quad \text{i.e.,} \quad \mathrm{J}_\ell=-\dot{u}_j\tau_{j\ell}. \tag{A2}$$

We consider elastic fields having a steady homogeneous component and a single frequency acoustic component. As usual, in bilinear expressions such as Eq. (A2), we turn to real parts of all quantities involved, which is consistent in a linear theory. Thus, we have

$$\dot{\boldsymbol{u}}(\boldsymbol{r},t)=\mathrm{Re}[-i\omega\boldsymbol{u}^0(\boldsymbol{r})\mathrm{e}^{-i\omega t}], \tag{A3}$$

$$\boldsymbol{\tau}(\boldsymbol{r},t)=\boldsymbol{\tau}^*+\mathrm{Re}[\delta\boldsymbol{\tau}^0(\boldsymbol{r})\mathrm{e}^{-i\omega t}]. \tag{A4}$$

Introducing this into Eq. (A2), we may single out the steady flow component of $\mathbf{J}$ by averaging over the period $2\pi/\omega$:

$$\langle\mathbf{J}\rangle=\tfrac{1}{2}(-i\omega\boldsymbol{u}^0\overline{\delta\boldsymbol{\tau}^0}+i\omega\overline{\boldsymbol{u}^0}\delta\boldsymbol{\tau}^0), \tag{A5}$$

where the overbar denotes complex conjugation. The time average $\langle W\rangle$ of the total elastic energy per unit length of the V region for a stationary irradiation is zero. Integration of the time average of Eq. (A1) then yields a balance equation for the steady time averaged energy flows across the surface of V:

$$\int_{C_\infty} dS\,\langle\mathbf{J}\rangle\cdot\hat{\boldsymbol{n}}=\int_S dS\,(-\langle\mathbf{J}\rangle)\cdot\hat{\boldsymbol{n}}, \tag{A6}$$

with $\hat{\boldsymbol{n}}$ the outer normal of V. With the sign conventions of Fig. 10, we obtain the following relation for the gain $\dot{W}_\infty$, defined as the imbalance between the incident and reflected energy flows:

$$\dot{W}_\infty=\langle\dot{W}_T\rangle+\langle\dot{W}_L\rangle-\langle\dot{W}_{inc}\rangle=\langle\dot{W}_{ext}\rangle, \tag{A7}$$

where

$$\langle\dot{W}_T\rangle+\langle\dot{W}_L\rangle=\langle\dot{W}_{refl}\rangle.$$

The energy flows associated with the plane wave beams crossing the dome C_∞ are easily obtained, if we use the well known steady current density for a plane wave [25],

$$|\langle\mathbf{J}\rangle_{L,T}|=\tfrac{1}{2}\rho\omega^2 c_{L,T}|U|^2,$$

valid for both polarizations; the quantity U is a possibly complex amplitude. The plane waves appearing in Eq. (A7) are defined in Eqs. (5) and (6). The coefficients α,β were obtained in Sec. III A. The three total energy flows are the products of current densities and beam cross sections. Thus,

$$\langle\dot{W}_{inc}\rangle=\tfrac{1}{2}\rho\omega^2 c_T\epsilon^2\left(\frac{\omega}{c_T q_T}\right)^2 S_0\frac{c_T q_T}{\omega},$$

$$\langle\dot{W}_T\rangle=\tfrac{1}{2}\rho\omega^2 c_T\epsilon^2|\alpha|^2\left(\frac{\omega}{c_T q_T}\right)^2 S_0\frac{c_T q_T}{\omega}, \tag{A8}$$

$$\langle\dot{W}_L\rangle=\tfrac{1}{2}\rho\omega^2 c_L\epsilon^2|\beta|^2\left(\frac{\omega}{c_L k}\right)^2 S_0\frac{c_L q_L}{\omega}.$$

The ratios of the flows $w_{TJ}=\langle\dot{W}_J\rangle/\langle\dot{W}_{inc}\rangle$ $(J=T,L)$ then have the form given in Eqs. (33) and (34) in the main text.

It remains to interpret the right-hand side $\langle\dot{W}_{ext}\rangle$ of Eq. (A7). First, using explicit expressions (A2) and (A5) for $\mathbf{J}$, $\langle\mathbf{J}\rangle$, we obtain [cf. Eq. (31)]

$$\langle\dot{W}_{ext}\rangle=\int_{S_0} dS\langle\dot{u}_1\tau_{12}\rangle, \tag{A9}$$

$$\langle\dot{W}_{ext}\rangle=\int_{S_0} dS\,\tfrac{1}{2}(-i\omega u_1^{(0)}\overline{\delta\tau_{12}^{(0)}}+i\omega\overline{u_1^{(0)}}\delta\tau_{12}^{(0)}). \tag{A10}$$

Equation (A10) may serve to check Eq. (A7) explicitly. Two points appear explicitly: (i) Only those irradiated parts of the interface where $\dot{u}_1 \neq 0$ do contribute; the effect is connected with the free sliding of the interface points along the sliding direction. (ii) All three waves superimposed enter $\langle \dot{W}_{ext} \rangle$. This is particularly remarkable in the case of an evanescent L wave, in which there is no longitudinal flow at infinity, yet the energy gain at the interface cannot be obtained correctly without including the L wave contribution right at the interface.

At each point of the interface, the external force acting on medium M is

$$\boldsymbol{F} = \boldsymbol{\tau} \cdot \hat{\boldsymbol{n}} = \boldsymbol{\tau}^* \cdot \hat{\boldsymbol{n}} + \mathrm{Re}(\delta \boldsymbol{\tau}) \cdot \hat{\boldsymbol{n}} \equiv \boldsymbol{F}^* + \delta \boldsymbol{F}.$$

The time average is

$$\langle \boldsymbol{F} \rangle = \boldsymbol{F}^*.$$

The averaged macroscopic force acting on S is thus $\mathcal{F}_S = S\boldsymbol{F}^*$. Medium M' is pulled at the overall velocity $\boldsymbol{v}$, thus the power spent by this force is $\mathcal{F}_S \cdot \boldsymbol{v}$, independently of the presence of oscillatory acoustic fields. It is now easy to see that

$$\mathcal{F}_S \cdot \boldsymbol{v} = \int_S dS \langle (\boldsymbol{v} - \dot{\boldsymbol{u}}) \cdot \boldsymbol{F} \rangle + \langle \dot{W}_{ext} \rangle. \tag{A11}$$

The first term is the power dissipated against the friction forces: $\boldsymbol{v} - \dot{\boldsymbol{u}}$ is the local relative interfacial velocity, while $\boldsymbol{F}$ has only a frictional component along the interface. Equation (A11) thus expresses the partitioning of the total work done per unit time by the external force into the (irreversibly) dissipated power and the net acoustic gain.

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